Depletion Approximation   Computational Algorithm

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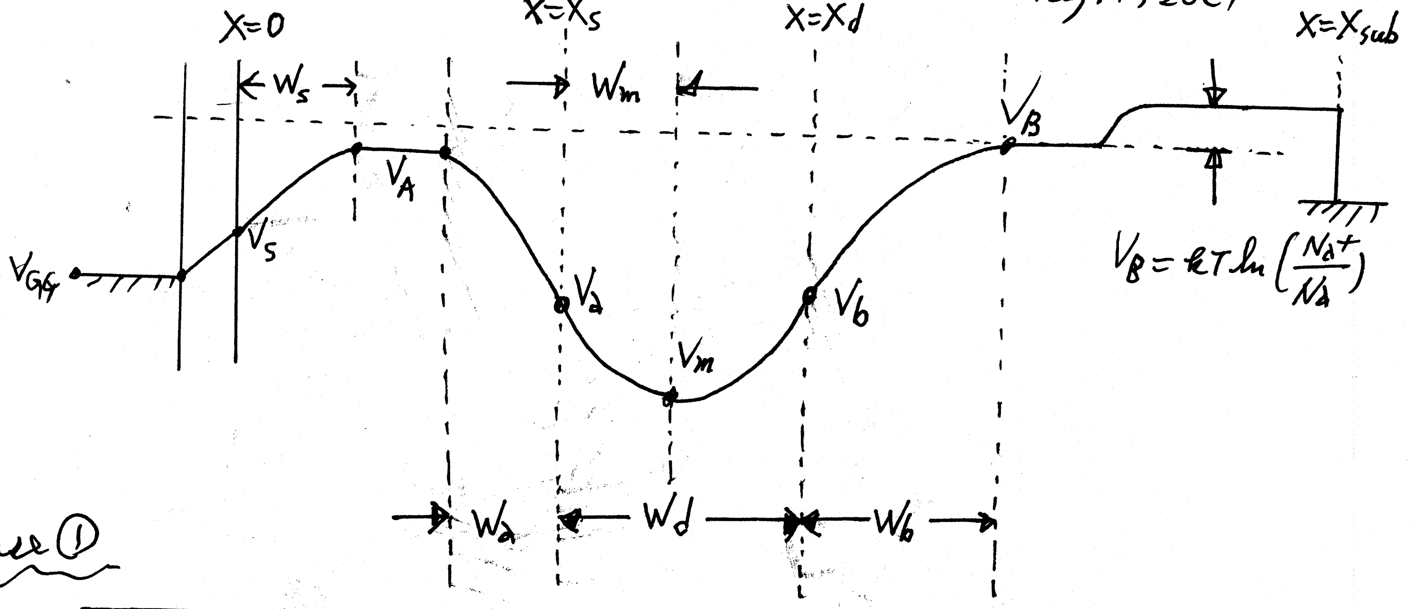
May 11, 2021



$V_B = kT \ln\left(\frac{N_a^+}{N_a}\right)$

Case 1

Choose W_m; $\quad W_a = \dfrac{N_d W_m}{N_a}$ — (1) $\quad W_b = \dfrac{N_d (W_d - W_m)}{N_a}$ — (2)

$V_a = V_A + \dfrac{N_a W_a^2}{2\varepsilon_{si}}$ — (3) $\quad V_b = V_B + \dfrac{N_a W_b^2}{2\varepsilon_{si}}$ — (4)

$V_{m1} = V_a + \dfrac{N_d W_m^2}{2\varepsilon_{si}}$ — (5) $\quad V_{m2} = V_b + \dfrac{N_d (W_d - W_m)^2}{2\varepsilon_{si}}$ — (6)

$\boxed{Error_1 = (V_{m1} - V_{m2})}$ — (7)

$V_{GG} = V_G + V_{FB} + Q_{ss} - Q_{sig}$; Choose V_s;

$E_s = \dfrac{C_o}{\varepsilon_{si}}(V_{GG} - V_s)$ — (8) $\quad W_s = \dfrac{\varepsilon_{si}}{N_a} E_s$ — (9)

$V_{s1} = V_A + \dfrac{N_a W_s^2}{2\varepsilon_{si}}$ — (10) $\quad \boxed{Error_2 = (V_{s1} - V_s)}$ — (11)

Check $W_s \leq (X_s - W_a)$ → yes, case 1

No

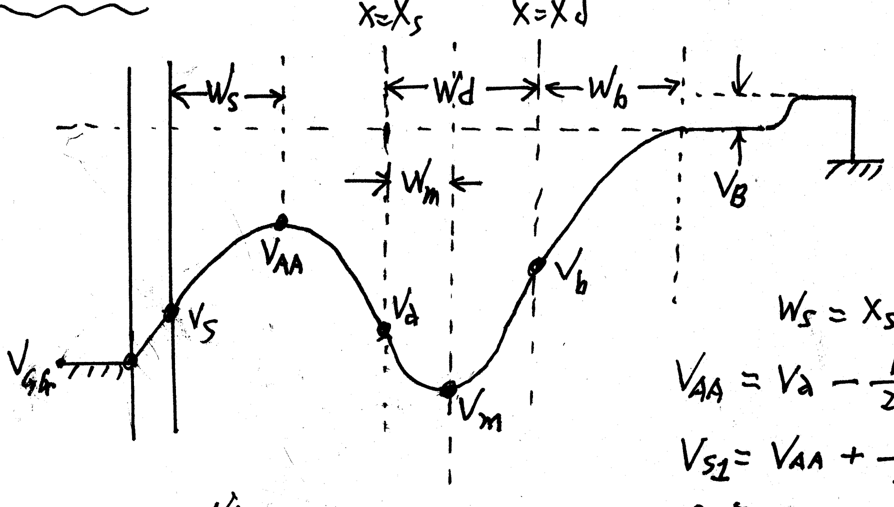
Case 2

Choose W_m $\quad W_b = \dfrac{N_d (W_d - W_m)}{N_a}$ — (12)

$V_b = V_B + \dfrac{N_a W_b^2}{2\varepsilon_{si}}$ — (13)

$V_m = V_b + \dfrac{N_d (W_d - W_m)^2}{2\varepsilon_{si}}$ — (14)

$V_a = V_m - \dfrac{N_d W_m^2}{2\varepsilon_{si}}$ — (15)



$W_s = X_s - \dfrac{N_d}{N_a} W_m$ — (16)

$V_{AA} = V_a - \dfrac{N_a}{2\varepsilon_{si}}(X_s - W_s)^2$ — (17)

$V_{s1} = V_{AA} + \dfrac{N_a}{2\varepsilon_{si}} W_s^2$ — (18)

$E_s = \dfrac{N_a}{\varepsilon_{si}} W_s$ — (19) $\quad V_{s2} = V_{GG} - \dfrac{\varepsilon_{si} E_s}{C_o}$ — (20) $\quad \boxed{Error_3 = (V_{s1} - V_{s2})}$ — (21)

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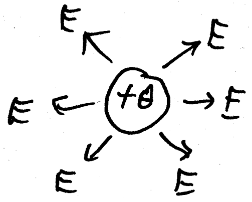
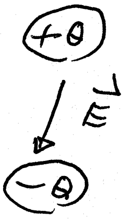
Gauss Law : the electric field flux is proportational to the charge inside.



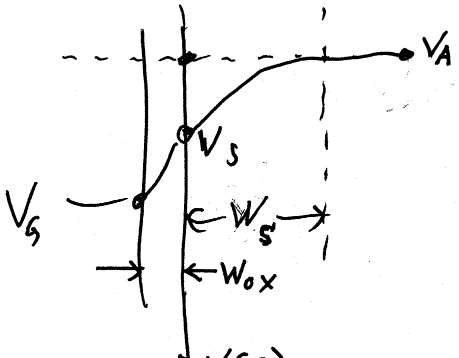
$$\boxed{\epsilon \int \vec{E} \cdot \vec{ds} = +Q}$$



(The electric field direction is from the positive charge $+Q$ to the negative charge $-Q$.)

The electric field $E(x)$ is defined as

$E(x) = -\frac{d}{dx} V(x)$ where $V(x)$ is the electron potential.



$V_G > V_S \quad \frac{dV}{dx} < 0 \quad E_S = -\frac{dV}{dx} > 0$ at $x = 0$.

$E_{ox} = \frac{(V_G - V_S)}{W_{ox}} > 0$ at inside oxide. —(1)

$\epsilon_{Si} E_S = \epsilon_{SiO_2} E_{ox} + (Q_{ss} - Q_{sig})$ —(2)

$E_S < E_{ox}$; (As the surface signal electrons Q_{sig} in the surface inversion region increases the surface electric field E_S and the surface potential both decrease.)

$V_S = V_A + \frac{N_A}{2\epsilon_{Si}} W_S^2$ —(3) $\quad E_S = \frac{N_A W_S}{\epsilon_{Si}}$ —(4)

If we write $C_o = \epsilon_{SiO_2}/W_{ox}$ we have $\epsilon_{Si} E_S = C_o (V_{GG} - V_S)$ —(5)

with $V_{GG} = V_G + V_{FB} + \frac{1}{C_o}(Q_{ss} - Q_{sig})$ —(6)

(3), (4) and (5) give $V_S = V_A + \frac{1}{2V_C}(V_{GG} - V_S)^2$ —(7) where $V_C = \frac{\epsilon_{Si} N_A}{C_o^2}$ —(8)

$$\left(\frac{V_S}{V_C}\right)^2 - 2\left(\frac{V_{GG}}{V_C} + 1\right)\left(\frac{V_S}{V_C}\right) + \left(\frac{V_{GG}}{V_C}\right)^2 + 2\left(\frac{V_A}{V_C}\right) = 0$$

$$\boxed{V_S = V_{GG} + V_C - \sqrt{V_C^2 + 2V_C(V_{GG} - V_A)}} \quad \text{—(9)}$$

[$\epsilon_{Si} \frac{d^2V}{dx^2} = \mathcal{D}(x) - N_A^+ \exp\left(-\frac{V}{kT}\right)$

with $V[0] = V_S$ at $x = 0$

$E_S = -\frac{dV}{dx}\Big|_{x=0}$ and $V[x] = 0$ at $x = X_{sub}$]

(at $X = X_{sub}$ $\mathcal{D}(x) = N_A^+ \gg N_A$

$\mathcal{D}(x)$ is an arbitrary function of X.)

Let the debye length be

$L_d = \sqrt{\frac{\epsilon_{Si} kT}{N_A^+}}$ —(10)

$$\boxed{\frac{d^2\left(\frac{V}{kT}\right)}{d\left(\frac{X}{L_d}\right)^2} = \left(\frac{\mathcal{D}(x)}{N_A^+}\right) - \exp\left(-\frac{V}{kT}\right)} \quad \text{—(11)}$$

We can normalize all equations with kT, L_d and N_A^+.

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After Normalization by kT, Ld and Na^+

$$\boxed{\frac{d^2V}{dx^2} = \mathcal{D}(x) - \exp(-V) \quad \text{with } V(0) = (h)\left.\frac{dV}{dx}\right|_{x=0} + V_{GG}}$$

(5) $\rightarrow$ $V_S = V_{GG} - \frac{\epsilon_{si}}{C_o} E_S$ (12)

$$\left(\frac{V_S}{kT}\right) = \left(\frac{\epsilon_{si}}{C_o}\right)\frac{d(V/kT)}{Ld \cdot d(\frac{x}{Ld})} + \frac{V_{GG}}{kT} \quad (13)$$

Let $h = \frac{\epsilon_{si}}{C_o L_d} = \left(\frac{\epsilon_{si}\ W_{ox}}{\epsilon_{SiO_2} L_d}\right)$ (14)

$$V[0] = \left(\frac{h}{dx}\right)(V[1] - V[0]) + V_{GG} \quad (14)$$

$$\left(\frac{h}{dx} + 1\right)V[0] = \left(\frac{h}{dx}\right)V[1] + V_{GG} \quad (15)$$

Let $V[0] = A[1]V[1] + B[1]$ (16)

Then we have

$$A[1] = \frac{h}{h + dx} \ (17); \quad B[1] = \frac{V_{GG}\,dx}{h + dx} \ (18);$$

$$\frac{d^2V}{dx^2} = \frac{V[i+1] + V[i-1] - 2V[i]}{(dx)^2} \ (19); \quad e^{-V} = e^{-V'}e^{V'-V} \approx e^{-V'}(1 + V' - V) \ (20);$$

Here we approximate $\mathcal{D}(x) - \exp(-V) \approx D[i] - \exp[-V'[i]](1 + V'[i] - V[i])$ (21)

Let $V[i+1] + V[i-1] - 2V[i] = EE[i]V[i] + HH[i]$ (22)

Then we have $HH[i] = (dx)^2[D[i] - \exp(-V'[i])(1 + V'[i])]$ (23)

and $EE[i] = (dx)^2 \exp[-V'[i]]$ (24)

> Let $V[i-1] = A[i]V[i] + B[i]$ for $i = N, (N-1), \dots, 1$. (25)
> Then we have $V[i] = A[i+1]V[i+1] + B[i]$ (26)
> and $V[i+1] = V[i]/A[i+1] - B[i+1]/A[i+1]$ (27)

From (22) then we have

$$V[i]/A[i+1] - B[i+1]/A[i+1] + A[i]V[i] + B[i] - 2V[i] = HH[i] + EE[i]V[i] \quad (28)$$

We then have

$$1/A[i+1] + A[i] - 2 = EE[i] \quad (29)$$

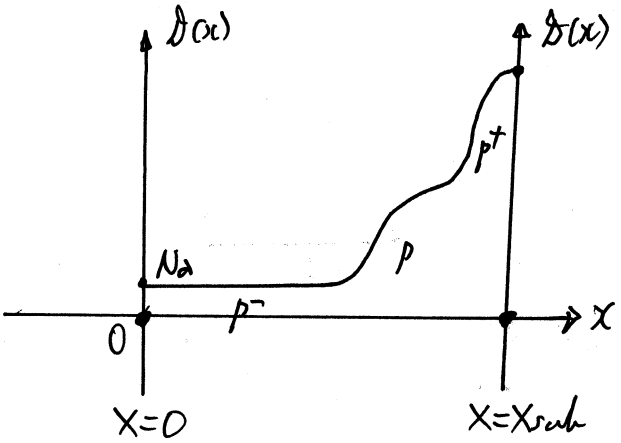
$$-B[i+1]/A[i+1] + B[i] = HH[i] \quad (30)$$

We finally get the iteration equations

> $A[i+1] = 1/(EE[i] - A[i] + 2)$
> $B[i+1] = A[i+1](B[i] - HH[i])$
> for $i = 1, 2, \dots, (N-1)$

> Then we have $V[N] = 0$ and
> $V[i-1] = A[i]V[i] + B[i]$
> for $i = N, N-1, \dots, 1$

The surface P^+P doping profile can be formed with the thermal diffusion and/or the ion implantation, and the buried N region can be formed by ion implantation with high energy. The doping profiles can be Gaussian or the Error functions type.



$$\begin{cases} \phi(0) = kT \ln\left(\frac{N_{aa}}{N_a}\right) \\ \phi(X_{sub}) = 0 \end{cases}$$

Solve for $\phi(x)$ for $0 < x < X_{sub}$

$$\epsilon_{si} \frac{d^2\phi}{dx^2} = D(x) - p(\phi)$$

$\left(p(\phi) = N_{aa} \exp(-\phi/kT) \text{ is the hole density.} \right)$

$\left(D(x) \text{ is an arbitrary doping profile. with } D(0) = N_a \text{ and } D(x_{sub}) = N_{aa} \right)$

$$d^2\phi = V[i+1] + V[i-1] - 2V[i] = (\epsilon_{si} dx^2)\left[D[i] - N_{aa} \exp\left(-\frac{V[i]}{kT}\right)\right]$$

<u>Method One</u> From the old values of $V[i]$, use the odd index values only, and compute new values of $V[i]$ from $V[i+1]$ and $V[i-1]$. (i = even) Then use the even values only to compute the new values of $V[i]$ (i = odd). Repeat the computation till the final values.

<u>Method Two</u> Let $V[i-1] = A[i]V[i] + B[i]$ for $i = N_p, (N_p - 1), \cdots, 2$.

We have $V[0] = V[1] = kT \ln\left(\frac{N_{aa}}{N_a}\right)$ and $V[N_p] = 0$.

Hence $A[1] = 1$ and $B[1] = 0$.

$$A[i+1]V[i+1] + B[i+1] = V[i]$$

$$V[i+1] = V[i]/A[i+1] - B[i+1]/A[i+1]$$

$$d^2\phi = V[i]/A[i+1] - B[i+1]/A[i+1] + A[i]V[i] + B[i] - 2V[i]$$
$$= (\epsilon_{si} dx^2)\left[D[i] - N_{aa} \exp\left[-\frac{V[i]}{kT}\right]\right]$$

Approximate $N_{aa} \exp\left(-\frac{V[i]}{kT}\right) \simeq N_{aa} \exp\left[-\frac{V_0[i]}{kT}\right]\left\{1 + \frac{V_0[i]}{kT} - \frac{V[i]}{kT}\right\}$

since $e^{-V} = e^{-V' + V' - V} = e^{-V'} e^{V' - V} \simeq e^{-V'}(1 + V' - V)$

<u>obtain</u> $\begin{cases} 1/A[i+1] + A[i] - 2 = (\epsilon_{si} dx^2) N_{aa} \exp\left[-\frac{V_0[i]}{kT}\right] \\ -B[i+1]/A[i+1] + B[i] = (\epsilon_{si} dx^2)\left(D[i] - N_{aa} \exp\left[-\frac{V_0[i]}{kT}\right]\left\{1 + \frac{V_0[i]}{kT}\right\}\right) \end{cases}$

For ($i = 2, 3, \cdots, N_p - 1$)

$$\begin{cases} A[i+1] = 1 / \left\{2 - A[i] + \epsilon_{si} dx^2 N_{aa} \exp\left(-\frac{V_0[i]}{kT}\right)\right\} \\ B[i+1] = A[i+1]\left\{B[i] - \epsilon_{si} dx^2 \left\{D[i] - N_{aa} \exp\left(-\frac{V_0[i]}{kT}\right)\left\{1 + \frac{V_0[i]}{kT}\right\}\right\}\right\} \end{cases}$$

<u>Then we have</u> $V[N_p] = 0$ and

$$V[i-1] = A[i]V[i] + B[i] \quad \text{for } (i = N_p, N_p - 1, \cdots, 1)$$