

中学の数学で解ける特殊相対性理論

後は、(10)式と(12)式は同じ物理現象(event)ですので同時に連立式として成り立ち、解くことが可能です。そして(13)式を求めることができます。(10)式と(12)式と(13)式を合わせてこれを Lorentz 変換と呼んでいます。特殊相対性理論の基本的な時空間ベクトル (t, x) の変換式となります。

Case(1) $x = 0$ $x = vt$ $x = ct$

Case(2) $x' = -vt'$ $x' = 0$ $x' = ct'$

Case(3) $x' = 0$ $x' = vt'$ $x = ct$

Case(1) and Case (2) describe the same event.

(10) For Case(1) and Case (2) event

$$x' = \beta (x - vt)$$

$$t' = \beta (t - vx/c^2)$$

(12) For Case(1) and Case (2) event

$$x = \beta (x' + vt')$$

$$t = \beta (t' + vx'/c^2)$$

Exchange A and B.

Case (3) is a different event from Case (1) and (2).

$$x' = \beta (\beta (x' + vt') - v \beta (t' + vx'/c^2)) = \beta^2 (1 - v^2/c^2) x'$$

$$t' = \beta (\beta (t' + vx'/c^2) - v \beta (x' + vt')/c^2) = \beta^2 (1 - v^2/c^2) t'$$

$$\beta^2 (1 - v^2/c^2) = 1 \tag{13}$$

$$\beta = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Case(1) と Case(2)は同じEvent. Case(3)は異なる。 .

Lorentz 変換

For Case(1) and Case (2) event

(10) $x' = \beta (x - vt)$
 $t' = \beta (t - vx/c^2)$

For Case(1) and Case (2) event

(12) $x = \beta (x' + vt')$
 $t = \beta (t' + vx'/c^2)$

For Case(3) event only,
not valid for Case (1) and (2)

(11) $x' = \beta (x + vt)$
 $t' = \beta (t + vx/c^2)$

(13) $\beta = \frac{1}{\sqrt{1 - v^2/c^2}}$

Case(1) $x = 0$ $x = vt$ $x = ct$

Case(2) $x' = -vt'$ $x' = 0$ $x' = ct'$

Case(3) $x' = 0$ $x' = vt'$ $x = ct$